

**PAPER-2 (B.E./B. TECH.)**

# **JEE (Main) 2020**

**COMPUTER BASED TEST (CBT)**

## **Memory Based Questions & Solutions**

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**Date: 09 January, 2020 (SHIFT-2) | TIME : (2.30 pm to 05.30 pm)**

**Duration: 3 Hours | Max. Marks: 300**

**SUBJECT : MATHEMATICS**

**PART : MATHEMATICS**

**SECTION – 1**

**Straight Objective Type (सीधे वस्तुनिष्ठ प्रकार)**

This section contains **19 multiple choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

इस खण्ड में **19 बहु-विकल्पी प्रश्न** हैं। प्रत्येक प्रश्न के 4 विकल्प (1), (2), (3) तथा (4) हैं, जिनमें से **सिर्फ एक सही** है।

1. If  $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$  then find the area bounded by  $f(x)$  and  $g(x)$  from  $x = \frac{1}{2}$  to  $x = \frac{\sqrt{3}}{2}$ .

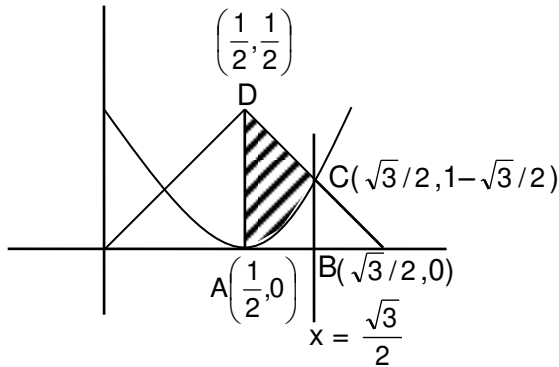
यदि  $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$  तब  $f(x)$  तथा  $g(x)$  के द्वारा  $x = \frac{1}{2}$  से  $x = \frac{\sqrt{3}}{2}$  तक परिबद्ध क्षेत्रफल ज्ञात कीजिये। .

- (1)  $\frac{\sqrt{3}}{4} - \frac{1}{3}$                       (2)  $\frac{\sqrt{3}}{4} + \frac{1}{3}$                       (3)  $2\sqrt{3}$                       (4)  $3\sqrt{3}$

**Ans. (1)**

**Sol.**



Required area = Area of trapezium ABCD -  $\int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

अभीष्ट क्षेत्रफल = समलम्ब चतुर्भुज ABCD का क्षेत्रफल -  $\int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

$$= \frac{1}{2} \left( \frac{\sqrt{3}-1}{2} \right) \left( \frac{1}{2} + 1 - \frac{\sqrt{3}}{2} \right) - \frac{1}{3} \left( \left( x - \frac{1}{2} \right)^3 \right)^{\frac{\sqrt{3}}{2}} \Big|_{\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

2.  $z$  is a complex number such that  $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$  then  $|z|$  can't be  
 $z$  एक समिश्र संख्या इस प्रकार है कि  $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$  तब  $|z|$  नहीं हो सकता

- (1)  $\sqrt{7}$                       (2)  $\sqrt{10}$                       (3)  $\sqrt{\frac{17}{2}}$                       (4)  $\sqrt{8}$

**Ans. (1)**

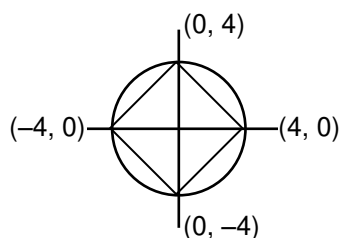
**Sol.**  $z = x + iy$   
 $|x| + |y| = 4$

Minimum value of  $|z| = 2\sqrt{2}$

Maximum value of  $|z| = 4$

$$|z| \in [\sqrt{8}, \sqrt{16}]$$

So  $|z|$  can't be  $\sqrt{7}$



**Sol.**  $z = x + iy$

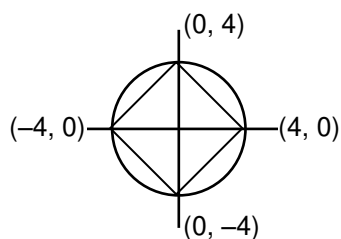
$$|x| + |y| = 4$$

$|z|$  का न्यूनतम मान  $= 2\sqrt{2}$

$|z|$  का अधिकतम मान  $= 4$

$$|z| \in [\sqrt{8}, \sqrt{16}]$$

अतः  $|z| = \sqrt{7}$  नहीं हो सकता



3. If यदि  $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$  and तथा  $a - 2b + c = 1$  then तब

- (1)  $f(50) = 1$  (2)  $f(-50) = -1$   
 (3)  $f(50) = 501$  (4)  $f(50) = -501$

**Ans.** (1)

**Sol.** Apply  $R_1 = R_1 + R_3 - 2R_2$  प्रयोग करने पर

$$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} \Rightarrow f(x) = 1 \Rightarrow f(50) = 1$$

4. Let  $a_n$  is a positive term of a GP and  $\sum_{n=1}^{100} a_{2n+1} = 200$ ,  $\sum_{n=1}^{100} a_{2n} = 100$  find  $\sum_{n=1}^{200} a_n$

माना  $a_n$  गुणोत्तर श्रेणी का धनात्मक पद है तथा  $\sum_{n=1}^{100} a_{2n+1} = 200$ ,  $\sum_{n=1}^{100} a_{2n} = 100$ ,  $\sum_{n=1}^{200} a_n$  का मान है-

- (1) 300 (2) 150 (3) 175 (4) 225

**Ans.** (2)

**Sol.** Let GP is  $a, ar, ar^2, \dots$

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \dots a_{201} = 200 \Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \quad \dots(1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots a_{200} = 100 = \frac{ar(r^{200}-1)}{r^2-1} = 100 \quad \dots(2)$$

Form (1) and (2)  $r = 2$

add both

$$\Rightarrow a_2 + a_3 + \dots a_{200} + a_{201} = 300 \Rightarrow r(a_1 + \dots a_{200}) = 300$$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

**Sol.** माना  $a, ar, ar^2, \dots$  गुणोत्तर श्रेणी में है

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \dots a_{201} = 200 \Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \quad \dots(1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \dots a_{200} = 100 = \frac{ar(r^{200}-1)}{r^2-1} = 100 \quad \dots(2)$$

समीकरण (1) तथा (2) से  $r = 2$

दोनों का योग करने पर

$$\Rightarrow a_2 + a_3 + \dots a_{200} + a_{201} = 300 \Rightarrow r(a_1 + \dots a_{200}) = 300$$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

5. If यदि  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ ,  $y(1) = 1$  and तथा  $y(x) = e$  then तब  $x = ?$

- (1)  $\frac{\sqrt{3}}{2} e$                       (2)  $\sqrt{3} e$                       (3)  $\sqrt{2} e$                       (4)  $\frac{e}{\sqrt{2}}$

**Ans.** (2)

**Sol.** Put  $y = vx$  रखने पर

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + x \frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2}$$

$$\Rightarrow \frac{1+v^2}{v^3} dv = -\frac{1}{x} dx$$

$$\Rightarrow \int \left( \frac{1}{v^3} + \frac{1}{v} \right) dv = \int \frac{-1}{x} dx$$

$$\Rightarrow \frac{-1}{2} \frac{1}{v^2} + \ln v = -\ln x + c$$

$$\Rightarrow -\frac{x^2}{2y^2} = -\ln y + c$$

When जब  $x = 1, y = 1$  then तब

$$-\frac{1}{2} = c$$

$$\Rightarrow x^2 = y^2(1 + 2\ln y)$$

$$\Rightarrow x^2 = e^2(3)$$

$$\Rightarrow x = \pm \sqrt{3} e$$

So इसलिये  $x = \sqrt{3}e$

6. Let probability distribution is

माना प्रायिकता वितरण इस प्रकार है

$x_i :$	1	2	3	4	5
$P_i :$	$k^2$	$2k$	$k$	$2k$	$5k^2$

then value of  $p(x > 2)$  is

तब  $p(x > 2)$  का मान है

- (1)  $\frac{7}{12}$                       (2)  $\frac{1}{36}$                       (3)  $\frac{1}{6}$                       (4)  $\frac{23}{36}$

**Ans.** (4)

**Sol.**  $\sum p_i = 1 \Rightarrow 6k^2 + 5k = 1$

$$6k^2 + 5k - 1 = 0$$

$$6k^2 + 6k - k - 1 = 0$$

$$(6k - 1)(k + 1) = 0 \Rightarrow k = -1 \text{ (rejected असत्य) ; } k = \frac{1}{6}$$

$$P(x > 2) = k + 2k + 5k^2 \\ = \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6+12+5}{36} = \frac{23}{36}$$

7.  $\int \frac{d\theta}{\cos^2 \theta (\sec 2\theta + \tan 2\theta)} = \lambda \tan \theta + 2 \log f(x) + c$  then ordered pair  $(\lambda, f(x))$  is

$$\int \frac{d\theta}{\cos^2 \theta (\sec 2\theta + \tan 2\theta)} = \lambda \tan \theta + 2 \log f(x) + c \text{ तब क्रमित युग्म } (\lambda, f(x)) \text{ है-}$$

- (1)  $(1, 1 + \tan \theta)$       (2)  $(1, 1 - \tan \theta)$       (3)  $(-1, 1 + \tan \theta)$       (4)  $(-1, 1 - \tan \theta)$

Ans.

(3)

Sol.

$$\int \frac{\sec^2 \theta}{\frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} + \frac{2 \tan \theta}{1 - \tan^2 \theta}} d\theta$$

$$= \int \frac{\sec^2 \theta (1 - \tan^2 \theta)}{(1 + \tan \theta)^2} d\theta$$

$$= \int \frac{\sec^2 \theta (1 - \tan \theta)}{1 + \tan \theta} d\theta$$

$$\tan \theta = t \Rightarrow \sec^2 \theta d\theta = dt$$

$$= \int \left( \frac{1-t}{1+t} \right) dt = \int \left( -1 + \frac{2}{1+t} \right) dt$$

$$= -t + 2 \log(1+t) + C$$

$$= -\tan \theta + 2 \log(1 + \tan \theta) + C$$

$$\Rightarrow \lambda = -1 \text{ and } f(x) = 1 + \tan \theta$$

8. If  $p \rightarrow (p \wedge \sim q)$  is false. Truth value of  $p$  &  $q$  will be

यदि  $p \rightarrow (p \wedge \sim q)$  असत्य है, तब  $p$  &  $q$  का सत्यता मान होगा

- (1) TT      (2) TF      (3) FT

(4) FF

Ans.

(1)

Sol.

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

9. If  $\lim_{x \rightarrow 0} x \left[ \frac{4}{x} \right] = A$  then the value of  $x$  at which  $f(x) = [x^2] \sin \pi x$  is discontinuous

(where  $[.]$  denotes greatest integer function)

यदि  $\lim_{x \rightarrow 0} x \left[ \frac{4}{x} \right] = A$  तब  $x$  का मान जिसके लिये  $f(x) = [x^2] \sin \pi x$  असतत् हो

(जहाँ  $[.]$  महत्तम पूर्णांक फलन को प्रदर्शित करता है)

- (1)  $\sqrt{A+1}$                       (2)  $\sqrt{A+21}$                       (3)  $\sqrt{A}$                       (4)  $\sqrt{A+5}$

**Ans.** (1)

**Sol.**  $\lim_{x \rightarrow 0} x \left( \frac{4}{x} - \left\{ \frac{4}{x} \right\} \right) = A \Rightarrow \lim_{x \rightarrow 0} 4 - x \left\{ \frac{4}{x} \right\} = A \Rightarrow 4 - 0 = A$

check when जाँचने पर

(A)  $x = \sqrt{A+1} \Rightarrow x = \sqrt{5} \Rightarrow$  discontinuous असतत्

(B)  $x = \sqrt{A+21} \Rightarrow x = 5 \Rightarrow$  continuous सतत्

(C)  $x = \sqrt{A} \Rightarrow x = 2 \Rightarrow$  continuous सतत्

(D)  $x = \sqrt{A+5} \Rightarrow x = 3 \Rightarrow$  continuous सतत्

10. Let one end of focal chord of parabola  $y^2 = 8x$  is  $\left( \frac{1}{2}, -2 \right)$ , then equation of tangent at other end of this focal chord is

माना परवलय  $y^2 = 8x$  की नाभिय जीवा के एक सिरे के निर्देशांक  $\left( \frac{1}{2}, -2 \right)$  है, तो दूसरे सिरे पर खींची गई स्पर्श रेखा

का समीकरण ज्ञात कीजिये।

- (1)  $x + 2y + 8 = 0$                       (2)  $x + 2y = 8$                       (3)  $x - 2y = 8$                       (4)  $x - 2y + 8 = 0$

**Ans.** (4)

**Sol.** Let माना  $\left( \frac{1}{2}, -2 \right)$  is  $(2t^2, 4t) \Rightarrow t = \frac{-1}{2}$

Parameter of other end of focal chord is 2

$\Rightarrow$  point is (8, 8)

$\Rightarrow$  equation of tangent is  $8y - 4(x+8) = 0$

$$\Rightarrow 2y - x = 8$$

नाभिय जीवा के दूसरे सिरे का प्राचल 2

$\Rightarrow$  अतः अन्य सिरे के निर्देशांक (8, 8) है

$\Rightarrow$  अतः स्पर्श रेखा का समीकरण  $8y - 4(x+8) = 0$

$$\Rightarrow 2y - x = 8$$

11. Let  $x + 6y = 8$  is tangent to standard ellipse where minor axis is  $\frac{4}{\sqrt{3}}$ , then eccentricity of ellipse is

माना  $x + 6y = 8$  मानक दीर्घवृत्त की स्पर्श रेखा का समीकरण है जिसकी लघु अक्ष की लंबाई  $\frac{4}{\sqrt{3}}$  है, तो दीर्घवृत्त की उत्केन्द्रता है

- (1)  $\sqrt{\frac{5}{6}}$                       (2)  $\sqrt{\frac{11}{12}}$                       (3)  $\frac{1}{3}\sqrt{\frac{11}{3}}$                       (4)  $\frac{1}{4}\sqrt{\frac{11}{12}}$

**Ans.** (2)

**Sol.**  $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

Equation of tangent  $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

स्पर्श रेखा का समीकरण  $\equiv y = mx \pm \sqrt{a^2m^2 + b^2}$

comparing with  $\equiv y = \frac{-x}{6} + \frac{4}{3}$

$\equiv y = \frac{-x}{6} + \frac{4}{3}$  से तुलना करने पर

$m = \frac{-1}{6}$  and तथा  $a^2m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} + \frac{4}{9} = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{9} = \frac{4}{9}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}}$

12. if  $f(x)$  and  $g(x)$  are continuous functions, fog is identity function,  $g'(b) = 5$  and  $g(b) = a$  then  $f'(a)$  is यदि  $f(x)$  तथा  $g(x)$  सतत् फलन है, fog तत्समक फलन है,  $g'(b) = 5$  तथा  $g(b) = a$  तब  $f'(a)$  है—

- (1)  $\frac{2}{5}$                       (2)  $\frac{1}{5}$                       (3)  $\frac{3}{5}$                       (4) 5

**Ans.** (2)

**Sol.**  $f(g(x)) = x$   
 $\Rightarrow f'(g(x)) \cdot g'(x) = 1$   
 Put  $x = b$   
 $\Rightarrow f'(g(b)) g'(b) = 1$   
 $\Rightarrow f'(a) \times 5 = 1$   
 $\Rightarrow f'(a) = \frac{1}{5}$



13. If यदि  $7x + 6y - 2z = 0$   
 $3x + 4y + 2z = 0$   
 $x - 2y - 6z = 0$  then which option is correct  
 $x - 2y - 6z = 0$  तब निम्न में से कौनसा विकल्प सत्य है—

- (1) no. solution (2) only trivial solution  
 (3) Infinite non trivial solution for  $x = 2z$  (4) Infinite non trivial solution for  $y = 2z$   
 (1) कोई हल नहीं (2) केवल तुच्छ हल  
 (3)  $x = 2z$  के लिए अनन्त अतुच्छ हल (4)  $y = 2z$  के लिये अनन्त अतुच्छ हल

Ans. (3)

Sol. 
$$\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$$= 7(-20) - 6(-20) - 2(-10)$$

$$= -140 + 120 + 20 = 0$$

so infinite non-trivial solution exist

अतः अनन्त अतुच्छ हल का अस्तित्व होगा

now equation (1) + 3 equation (3)

अब समीकरण (1) + 3 समीकरण (3)

$$10x - 20z = 0$$

$$x = 2z$$

14. Let  $x = 2\sin\theta - \sin 2\theta$  and  
 $y = 2\cos\theta - \cos 2\theta$

find  $\frac{d^2y}{dx^2}$  at  $\theta = \pi$

माना  $x = 2\sin\theta - \sin 2\theta$  तथा

$$y = 2\cos\theta - \cos 2\theta$$

$\theta = \pi$  पर  $\frac{d^2y}{dx^2}$  का मान ज्ञात कीजिये।

(1)  $\frac{3}{8}$

(2)  $\frac{3}{2}$

(3)  $\frac{5}{8}$

(4)  $\frac{7}{8}$

Sol.  $\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\therefore \frac{dy}{dx} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta}$$

$$= \frac{2\sin\frac{\theta}{2} \cdot \cos\frac{3\theta}{2}}{2\sin\frac{\theta}{2} \sin\frac{3\theta}{2}} = \cot\frac{3\theta}{2}$$



$\alpha = \frac{b}{a}$  is also root of  $x^2 - 2bx - 10 = 0$

$\alpha = \frac{b}{a}$ ,  $x^2 - 2bx - 10 = 0$  का मूल है

$$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$$

by (i) से  $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$$\Rightarrow 20a^2 = 5a$$

$$\Rightarrow a = \frac{1}{4} \text{ and तथा } b^2 = \frac{5}{4}$$

$$\alpha^2 = 20 \text{ and तथा } \beta^2 = 5$$

Now अब  $\alpha^2 + \beta^2$

$$= 5 + 20$$

$$= 25$$

17. Let  $A = \{x : |x| < 2\}$  and  $B = \{x : |x - 2| \geq 3\}$  then

माना  $A = \{x : |x| < 2\}$  तथा  $B = \{x : |x - 2| \geq 3\}$  तब

(1)  $A \cap B = [-2, -1]$

(2)  $A \cup B = \mathbb{R} - (2, 5)$

(3)  $A - B = [-1, 2)$

(4)  $B - A = \mathbb{R} - (-2, 5)$

Ans. (4)

Sol.

$$A = \{x : x \in (-2, 2)\}$$

$$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$$

$$A \cap B = \{x : x \in (-2, -1]\}$$

$$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$$

$$A - B = \{x : x \in (-1, 2)\}$$

$$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$$

18. Let  $x = \sum_{n=0}^{\infty} (-1)^n (\tan \theta)^{2n}$  and  $y = \sum_{n=0}^{\infty} (\cos \theta)^{2n}$  where  $\theta \in (0, \pi/4)$ , then

माना  $x = \sum_{n=0}^{\infty} (-1)^n (\tan \theta)^{2n}$  तथा  $y = \sum_{n=0}^{\infty} (\cos \theta)^{2n}$  जहाँ  $\theta \in (0, \pi/4)$ , तब

(1)  $x(y + 1) = 1$

(2)  $y(1 - x) = 1$

(3)  $y(x - 1) = 1$

(4)  $y(1 + x) = 1$

Ans. (2)

Sol.

$$y = 1 + \cos^2 \theta + \cos^4 \theta + \dots$$

$$\Rightarrow y = \frac{1}{1 - \cos^2 \theta} \Rightarrow \frac{1}{y} = \sin^2 \theta$$

$$x = \frac{1}{1 - (-\tan^2 \theta)} = \frac{1}{\sec^2 \theta}$$

$$\Rightarrow \cos^2 \theta = x$$

$$\Rightarrow \frac{1}{y} + x = 1$$

$$\Rightarrow y(1 - x) = 1$$

19. Let the distance between plane passing through lines  $\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+1}{8} = 8$  and  $\frac{x+3}{2} =$

$\frac{y+2}{1} = \frac{z-1}{\lambda}$  and plane  $23x - 10y - 2z + 48 = 0$  is  $\frac{k}{\sqrt{633}}$  then k is equal to

माना सरल रेखाओं  $\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+1}{8} = 8$  तथा  $\frac{x+3}{2} = \frac{y+2}{1} = \frac{z-1}{\lambda}$  से गुजरने वाले समतल तथा समतल

$23x - 10y - 2z + 48 = 0$  के मध्य दूरी  $\frac{k}{\sqrt{633}}$  है तब k का मान है-

(1) 1

(2) 2

(3) 3

(4) 4

Ans. (3)

Sol. Lines must be intersecting

$$\Rightarrow (2s - 1, 3s + 3, 8s - 1) = (2t - 3, t - 2, \lambda t + 1)$$

$$2s - 1 = 2t - 3, 3s + 3 = t - 2, 8s - 1 = \lambda t + 1 \quad \Rightarrow t = -1, s = -2, \lambda = 18$$

distance of plane contains given lines from given plane is same as distance between point  $(-3, -2, 1)$  from given plane.

$$\text{Required distance equal to } \frac{|-69 + 20 - 2 + 48|}{\sqrt{529 + 100 + 4}} = \frac{3}{\sqrt{633}} = \frac{k}{\sqrt{633}} \Rightarrow k = 3$$

Sol. रेखाएँ प्रतिच्छेदी होगी

$$\Rightarrow (2s - 1, 3s + 3, 8s - 1) = (2t - 3, t - 2, \lambda t + 1)$$

$$2s - 1 = 2t - 3, 3s + 3 = t - 2, 8s - 1 = \lambda t + 1 \quad \Rightarrow t = -1, s = -2, \lambda = 18$$

रेखाओं का समाहित करने वाले समतल की दिये हुये समतल से दूरी = बिंदु  $(-3, -2, 1)$  से दिये गये समतल की दूरी

$$\text{अभीष्ट दूरी} = \frac{|-69 + 20 - 2 + 48|}{\sqrt{529 + 100 + 4}} = \frac{3}{\sqrt{633}} = \frac{k}{\sqrt{633}} \Rightarrow k = 3$$

## SECTION - 2

- ❖ This section contains **FOUR (04)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- ❖ If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If **ONLY** the correct option is chosen.
  - Zero Marks : **0** In all other cases

### खंड 2

- ❖ इस खंड में **चार (04)** प्रश्न हैं। प्रत्येक प्रश्न का उत्तर संख्यात्मक मान (**NUMERICAL VALUE**) हैं, जो द्वि-अंकीय पूर्णांक तथा दशमलव एकल-अंकन में है।
- ❖ यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है, तो संख्यात्मक मान को दशमलव के दो स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।
- ❖ अंकन योजना :
  - पूर्ण अंक : **+4** यदि सिर्फ सही विकल्प ही चुना गया है।
  - शून्य अंक : **0** अन्य सभी परिस्थितियों में।

20. If  ${}^{25}C_0 + 5 {}^{25}C_1 + 9 {}^{25}C_2 + \dots + 101 {}^{25}C_{25} = 2^{25} k$  find  $k = ?$   
यदि  ${}^{25}C_0 + 5 {}^{25}C_1 + 9 {}^{25}C_2 + \dots + 101 {}^{25}C_{25} = 2^{25} k$  तब  $k = ?$

Ans. (51)

**Sol.** 
$$\sum_{r=0}^{25} (4r+1)^{25} C_r = 4 \sum_{r=0}^{25} r \cdot 25 C_r + \sum_{r=0}^{25} 25 C_r$$

$$= 4 \sum_{r=1}^{25} r \times \frac{25}{r} {}^{24}C_{r-1} + 2^{25} = 100 \sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25}$$

$$= 100 \cdot 2^{24} + 2^{25} = 2^{25}(50 + 1) = 51 \cdot 2^{25}$$
 So इसलिये  $k = 51$

**21.** Let circles  $(x - 0)^2 + (y - 4)^2 = k$  and  $(x - 3)^2 + (y - 0)^2 = 1^2$  touches each other than find the maximum value of 'k'  
 माना वृत्त  $(x - 0)^2 + (y - 4)^2 = k$  तथा  $(x - 3)^2 + (y - 0)^2 = 1^2$  एक दूसरे को स्पर्श करते है तब 'k' का अधिकतम मान ज्ञात कीजिये।

**Ans. 36.00**

**Sol.** Two circles touches each other if  $C_1 C_2 = |r_1 \pm r_2|$

Distance between  $C_2(3, 0)$  and  $C_1(0, 4)$  is either  $\sqrt{k} + 1$  or  $|\sqrt{k} - 1|$  ( $C_1 C_2 = 5$ )

$\Rightarrow \sqrt{k} + 1 = 5$  or  $|\sqrt{k} - 1| = 5 \Rightarrow k = 16$  or  $k = 36 \Rightarrow$  maximum value of k is 36

**Sol.** दो वृत्त एक दूसरे को स्पर्श करते हैं यदि  $C_1 C_2 = |r_1 \pm r_2|$

$C_2(3, 0)$  तथा  $C_1(0, 4)$  के मध्य दूरी  $\sqrt{k} + 1$  या  $|\sqrt{k} - 1|$  है ( $C_1 C_2 = 5$ )

$\Rightarrow \sqrt{k} + 1 = 5$  or  $|\sqrt{k} - 1| = 5 \Rightarrow k = 16$  or  $k = 36 \Rightarrow$  k का अधिकतम मान 36 है

**22.** Let  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 5$ ,  $\vec{b} \cdot \vec{c} = 10$  angle between  $\vec{b}$  &  $\vec{c}$  equal to  $\frac{\pi}{3}$

If  $\vec{a}$  is perpendicular to  $\vec{b} \times \vec{c}$  then find the value of  $|\vec{a} \times (\vec{b} \times \vec{c})|$

माना  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 5$ ,  $\vec{b} \cdot \vec{c} = 10$ ,  $\vec{b}$  &  $\vec{c}$  के बीच का कोण  $\frac{\pi}{3}$  के बराबर है।

यदि  $\vec{a}$ ,  $\vec{b} \times \vec{c}$  के लम्बवत् है तब  $|\vec{a} \times (\vec{b} \times \vec{c})|$  का मान ज्ञात कीजिये।

**Ans. 30**

**Sol.**  $\vec{b} \cdot \vec{c} = 10 \Rightarrow |\vec{b}| |\vec{c}| \cos\left(\frac{\pi}{3}\right) = 10 \Rightarrow 5 \cdot |\vec{c}| \cdot \frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$

Also तथा,  $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

$$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}| |\vec{b} \times \vec{c}| \sin\left(\frac{\pi}{2}\right)$$

$$\sqrt{3} \times |\vec{b}| |\vec{c}| \sin\frac{\pi}{3} \times 1 = \sqrt{3} \times 5 \times 4 \times \frac{\sqrt{3}}{2} = 30$$

**23.** Number of common terms in both sequence 3, 7, 11, .....407 and 2, 9, 16, .....905 is  
 श्रेणियों 3, 7, 11, .....407 तथा 2, 9, 16, .....905 में उभयनिष्ठ पदों की संख्या है—

**Ans. (14)**

**Sol.** First common term = 23  
 common difference =  $7 \times 4 = 28$   
 Last term  $\leq 407$   
 $\Rightarrow 23 + (n-1) \times 28 \leq 407$   
 $\Rightarrow (n-1) \times 28 \leq 384$   
 $\Rightarrow n \leq 13.71 + 1$   
 $n \leq 14.71$   
 So  $n = 14$

**Sol.** प्रथम उभयनिष्ठ पद = 23  
 सार्व अंतर =  $7 \times 4 = 28$   
 अंतिम पद  $\leq 407$   
 $\Rightarrow 23 + (n-1) \times 28 \leq 407$   
 $\Rightarrow (n-1) \times 28 \leq 384$   
 $\Rightarrow n \leq 13.71 + 1$   
 $n \leq 14.71$   
 इसलिये  $n = 14$

**24.** If minimum value of term free from x for  $\left(\frac{x}{\sin\theta} + \frac{1}{x\cos\theta}\right)^{16}$  is  $L_1$  in  $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$  and  $L_2$  in  $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$  find

$$\frac{L_2}{L_1}$$

यदि  $\left(\frac{x}{\sin\theta} + \frac{1}{x\cos\theta}\right)^{16}$  में x रहित पद का न्यूनतम मान  $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$  के लिए  $L_1$  है तथा  $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$  के लिये  $L_2$

है तब  $\frac{L_2}{L_1}$  का मान ज्ञात कीजिये।

**Ans.** 16

**Sol.**  $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^r$

for  $r = 8$  term is free from 'x'

$$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta \cos^8\theta}$$

$$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$$

$$\text{in } \theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right], L_1 = {}^{16}C_8 \cdot 2^8 \quad \because \{\text{Min value of } L_1 \text{ at } \theta = \pi/4\}$$

$$\text{in } \theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right], L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 \cdot 2^8 \cdot 2^4 \quad \because \{\text{min value of } L_2 \text{ at } \theta = \pi/8\}$$

$$\frac{L_2}{L_1} = \frac{{}^{16}C_8 \cdot 2^8 \cdot 2^4}{{}^{16}C_8 \cdot 2^8} = 16$$

**Sol.**  $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^r$

$r = 8$  से 'x' रहित पद

$$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta \cos^8\theta}$$

$$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta \cos^8\theta}$$

$$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$$

$$\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right] \text{ में, } L_1 = {}^{16}C_8 2^8 \quad \because \{L_1 \text{ का न्यूनतम मान } \theta = \pi/4 \text{ पर}\}$$

$$\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right] \text{ में, } L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 \cdot 2^8 \cdot 2^4 \quad \{\because L_2 \text{ का न्यूनतम मान } \theta = \pi/8\}$$

$$\frac{L_2}{L_1} = \frac{{}^{16}C_8 \cdot 2^8 \cdot 2^4}{{}^{16}C_8 \cdot 2^8} = 16$$